Problem Solving

Solve this maze at your leisure

Start at Phil's house. At first, you can only make *right* turns through the maze. Each time you cross the red zigzag sign (under Carl’s auto repair), the direction in which you turn changes. So, after the first time you cross that sign, you can then only make *left* turns; after the second time, you switch back to *right* turns only, etc. How can Carl’s auto repair be reached?

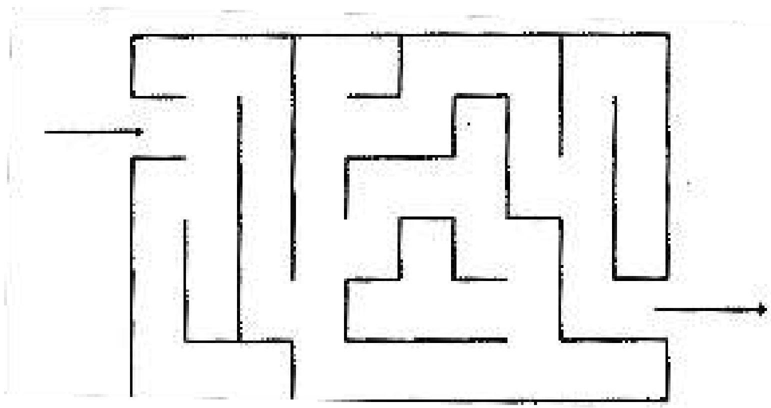
Views of Problem solving

- **Well-defined problems**
  - Much studied in AI
  - Requires search
  - Domain general heuristics for solving problems

- What about **ill-defined** problems?
  - No real mechanisms for dealing with these
  - The problem may be solved suddenly by ‘seeing’ the problem differently
  - Often requires developing a **suitable representation**

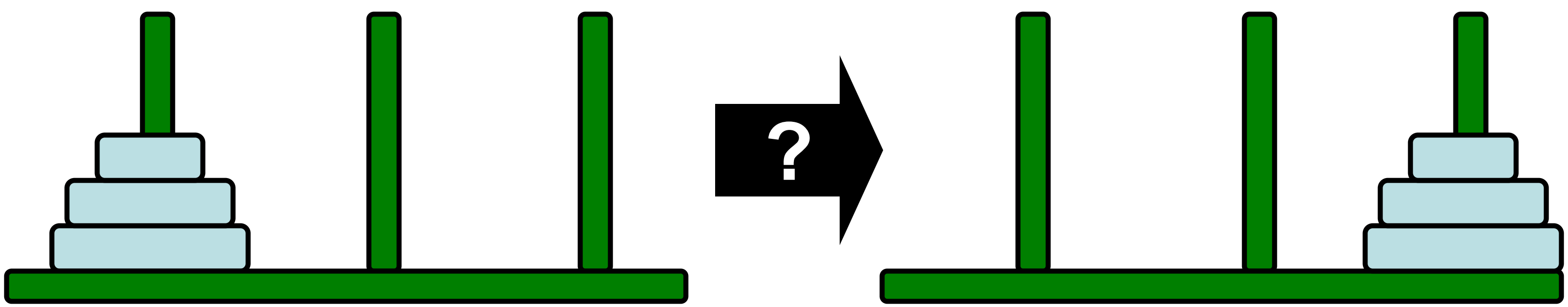
Problem solving as search

INITIAL STATE



GOAL STATE

INITIAL STATE GOAL STATE

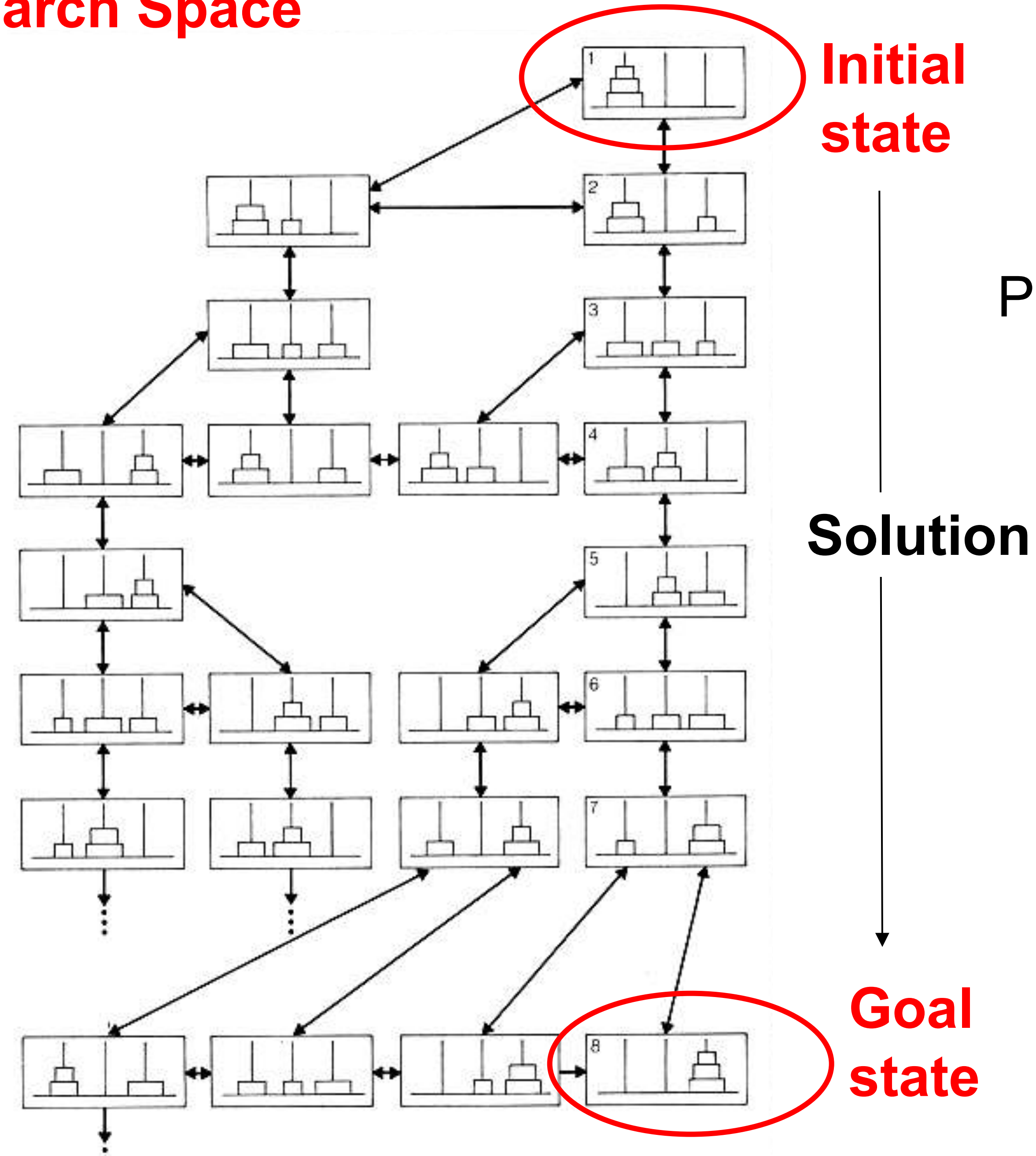


Play the game: http://www.mazeworks.com/hanoi/

Solving most games involves search

- Examples:
  - Cannibals and missionaries:
    http://www.learn4good.com/games/puzzle/boat.htm
  - Theseus and the Minotaur:
    http://www.logicmazes.com/theseus.html
  - More special mazes
    http://www.logicmazes.com/

Search Space



Problem Solving is *a search problem*

Today, the main theory underlying research on problem solving is ***problem space theory***, developed by Allen Newell and Herbert Simon and described in their book, *Human Problem Solving* (1972). This led to an **information processing theory** of problem solving, viewing problem-solving as a serial search process.

Problem solving, in this view, is a search within a **problem space**, which is the set of states, or possible choices, that faces the problem solver at each step in moving from an initial state to a goal state.

The problem space includes the initial state, the goal state, and all possible intermediate states.

Search spaces can be large

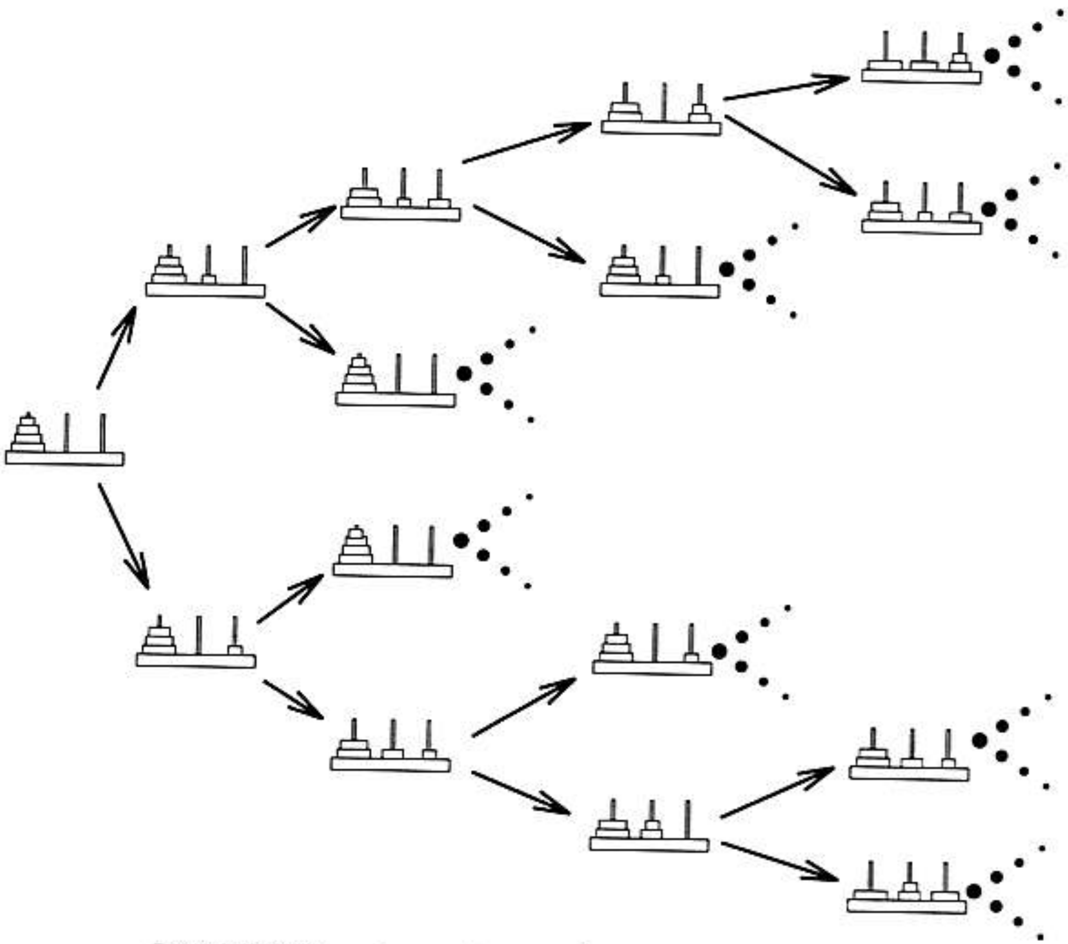


FIGURE 3 A search space for the Tower of Hanoi.

#DISCS	#STATES
3	$3^3 = 27$
4	$3^4 = 81$
5	$3^5 = 243$
6	$3^6 = 729$

What if the search space is too large?

- It is not possible to enumerate the entire search space for many well-defined problems

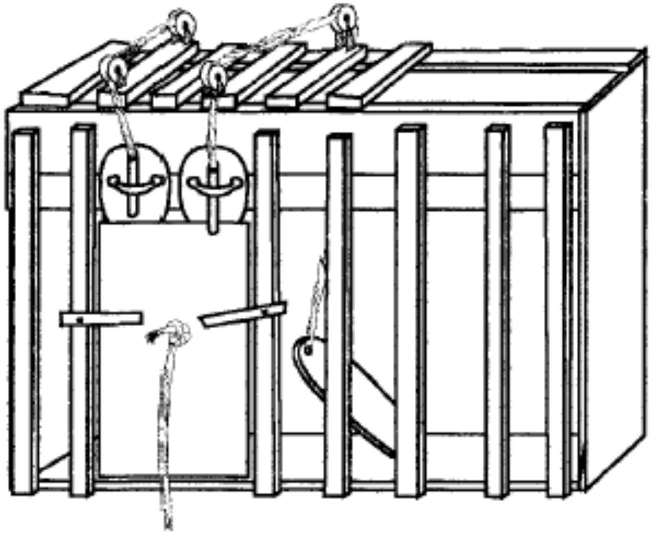
- We must use **heuristics**
  - Not guaranteed to work but easy to implement
  - Example heuristics
    - ✓ **Trial and error (反复试验，试错法)**
    - ✓ **Hill climbing (爬山法)**
    - ✓ **Means-end analysis (手段目的分析)**

Trial and Error

- Edward L. Thorndike (1874-1949) found that many animals search by **trial and error** (aka **random search**)

- Found that cats in a "puzzle box" (see left) initially behaved impulsively and apparently random

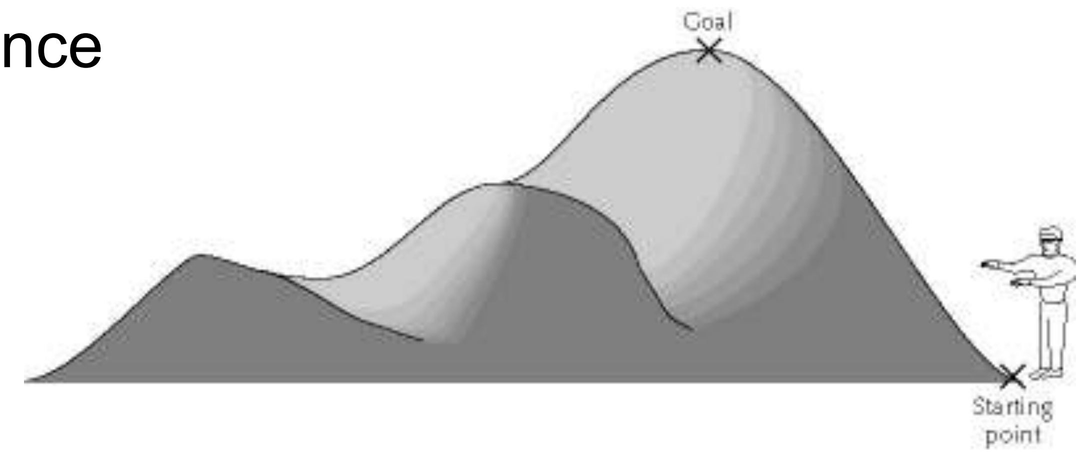
- After many trials in puzzle box, solution time decreases

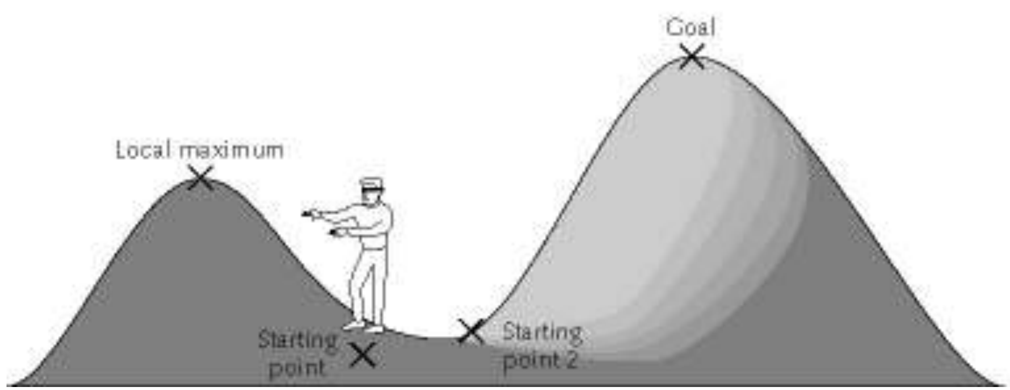


In order to escape the animal has to perform three different actions: press a pedal, pull on a string, and push a bar up or down

Hill Climbing

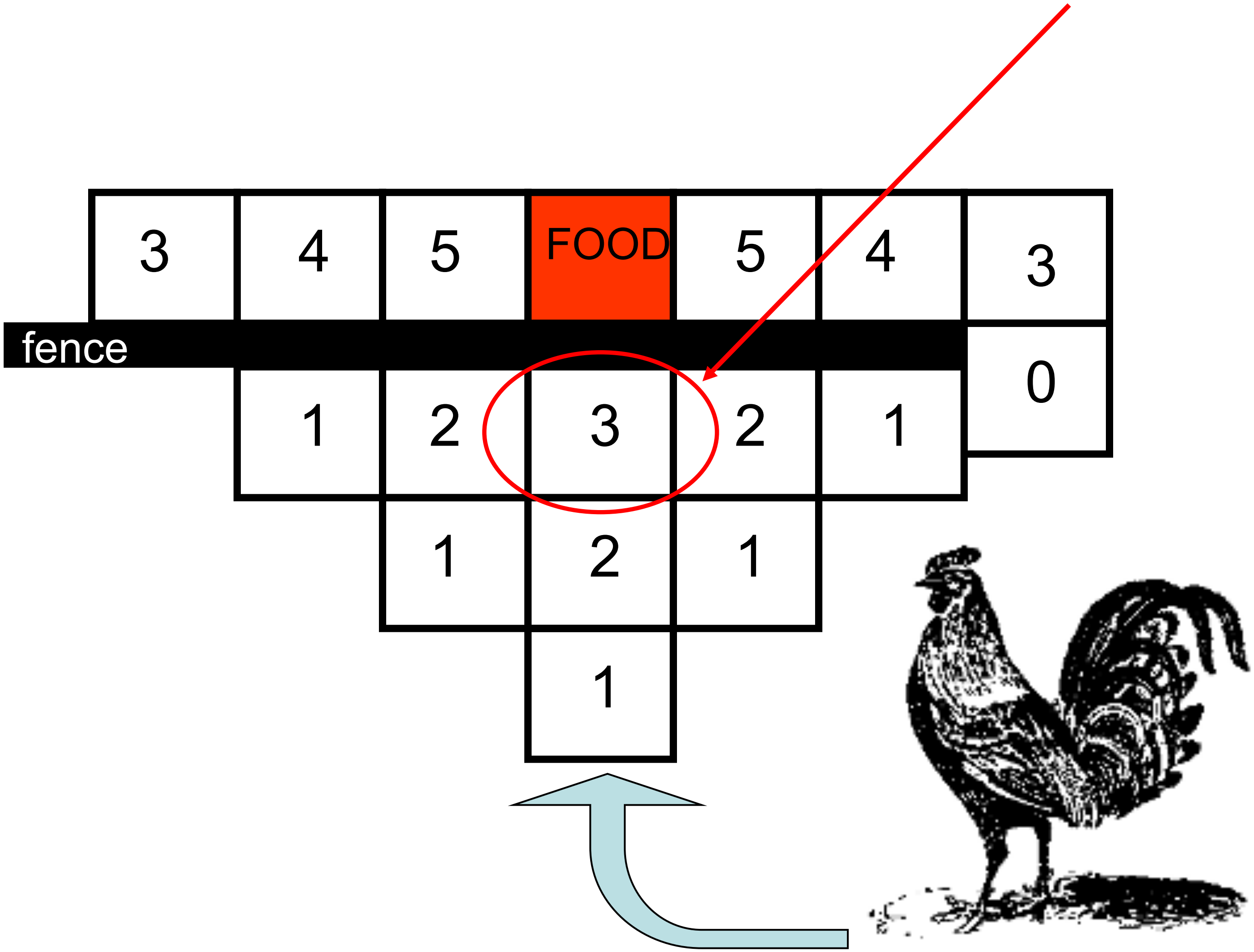
- Find some measure of the distance between your present state and the end state
  - ➢ Take a step in the direction that most reduces that distance





Hill Climbing

- Might lead to suboptimal solutions: local maximum



Hill Climbing

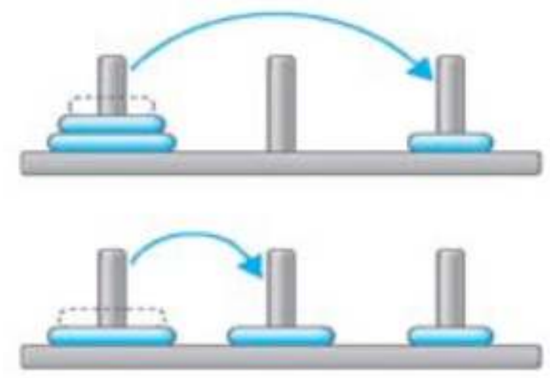
- According to the hill-climbing strategy, at each step, you choose a path that moves you closer to the goal. This strategy can fail when steps are required that initially will move away from the goal

- Prefrontal cortex is required for planning moves, which is necessary to avoid (“escape”) suboptimal states

- Clinical cases of perseverance (坚持不懈，不屈不挠); patients with frontal lobe damage repeat same move over and over

Means-end analysis

- Set up a goal
- Look for a difference between current state and goal or subgoal state
- Find an **operator** to reduce this difference. One operator is the setting of a new **subgoal**
- Apply operator
- Repeat until final goal is achieved



(a) Subgoal 1: Free up large disc.

(b) Subgoal 2: Free up third peg.

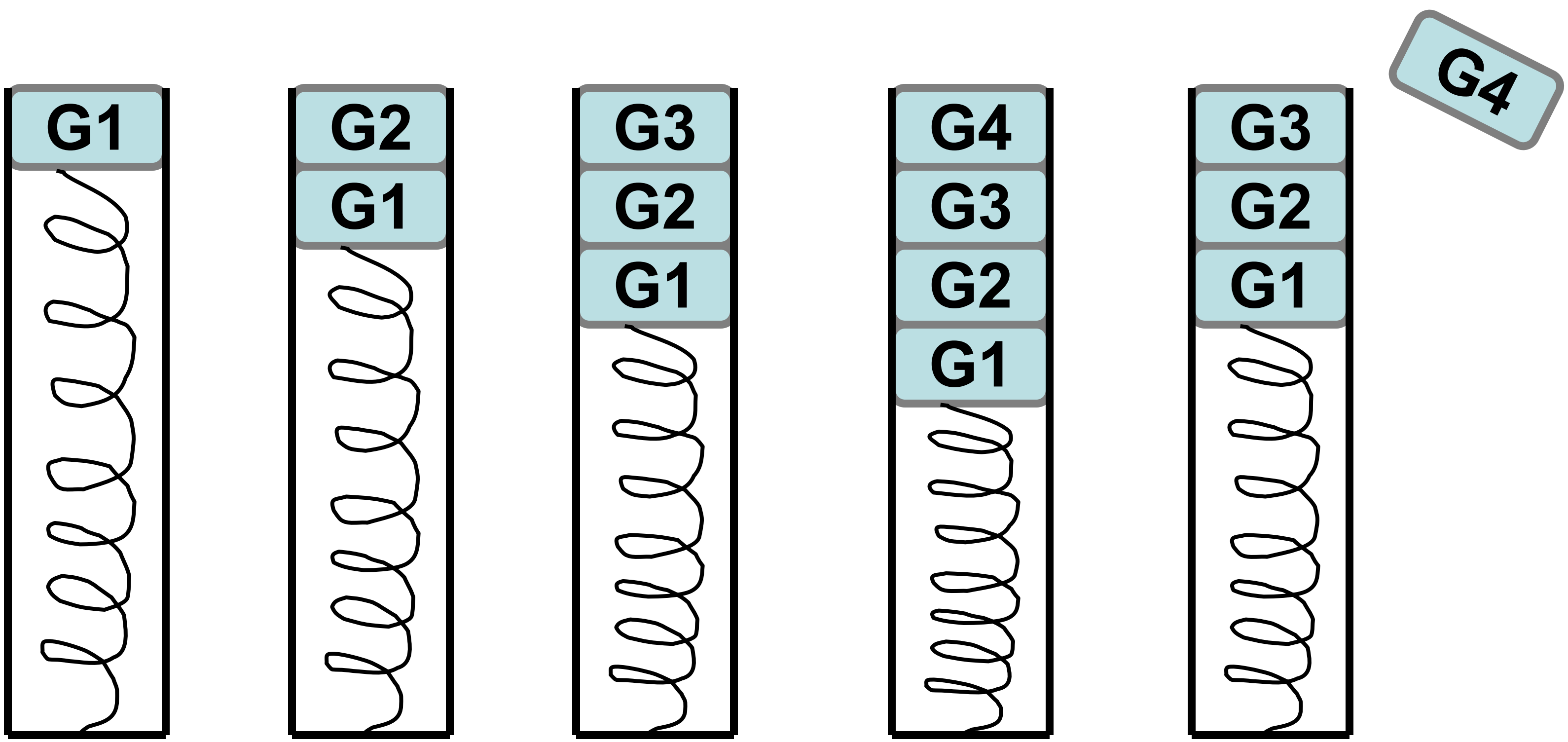
(c) Subgoal 3: Move large disc onto third peg.

Setting subgoals in means-end analysis

• Painting your house	(GOAL 1)
• Apply paint	(SUBGOAL 2)
• Need paint and brush	(SUBGOAL 3)
• Go to hardware store	(SUBGOAL 4)
• Went to hardware store	(SUBGOAL 4)
• Got paint and brush	(SUBGOAL 3)
• Apply paint	(SUBGOAL 2)
• Paint the house	(GOAL 1)

Goal Stack: last in, first out

Push Goal 1 on Stack	Push Goal 2 on Stack	Push Goal 3 on Stack	Push Goal 4 on Stack	Solved Goal 4: Pop-off Stack



What about ill-defined problems?

- No real mechanisms for dealing with these

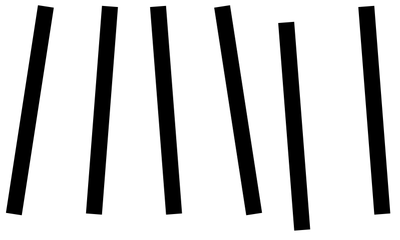
- According to Gestalt psychologists, the problem may be solved suddenly by ‘seeing’ the problem differently

- Often requires developing a suitable representation

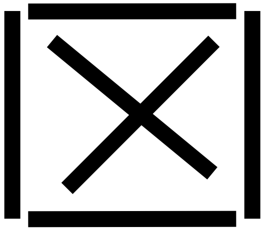
A special case of an ill-defined problem is known as the **insight problem** （顿悟问题）, to which, despite all the unknowns, the answer seems to come all of a sudden in a flash of understanding

Six stick problem

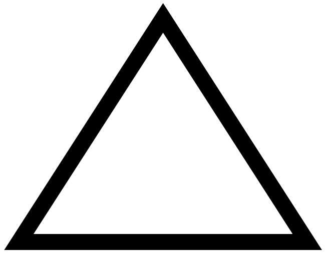
With these six sticks:



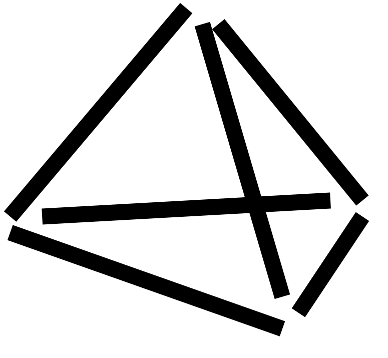
Wrong solution:



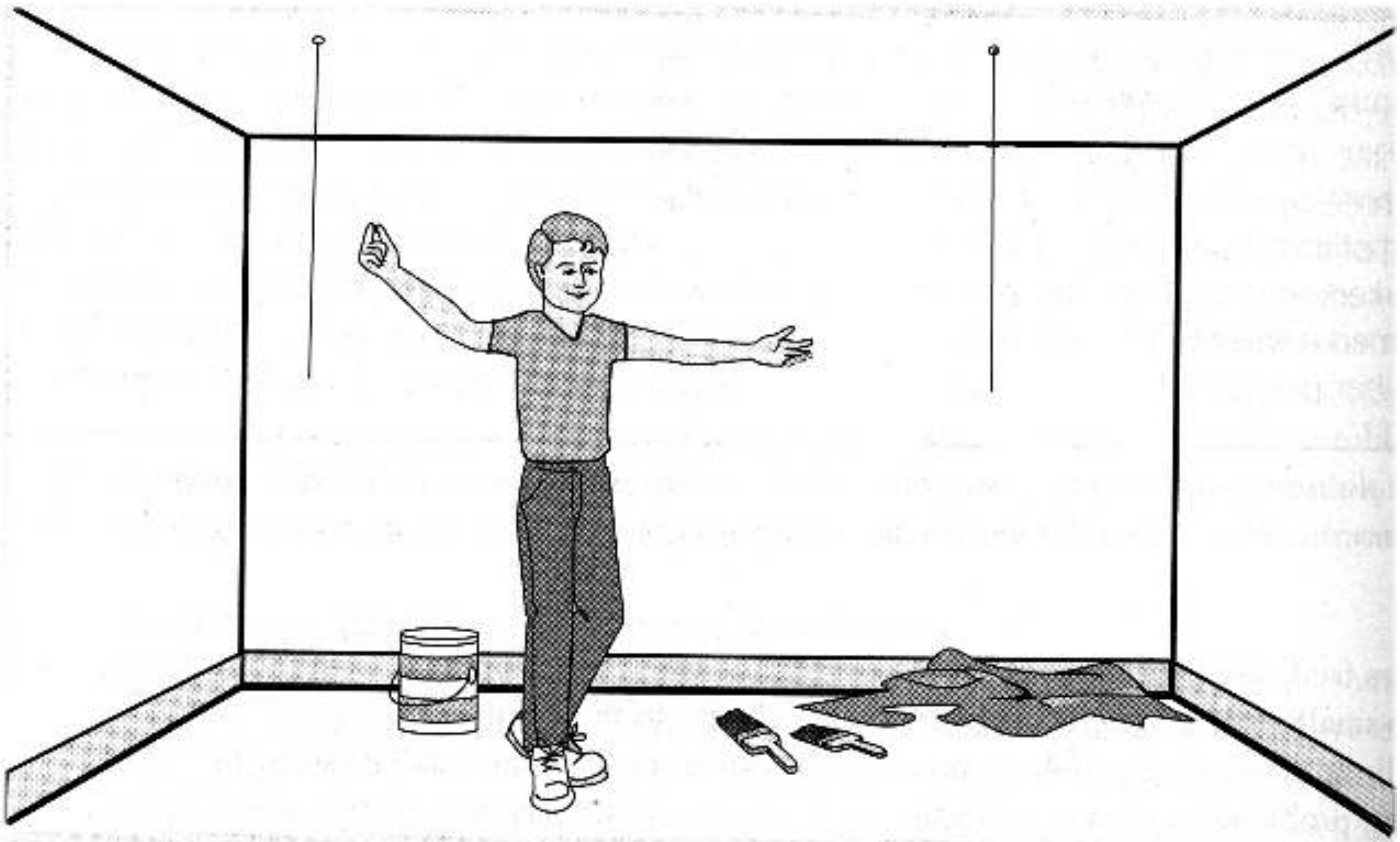
Make four equilateral triangles:



Answer:

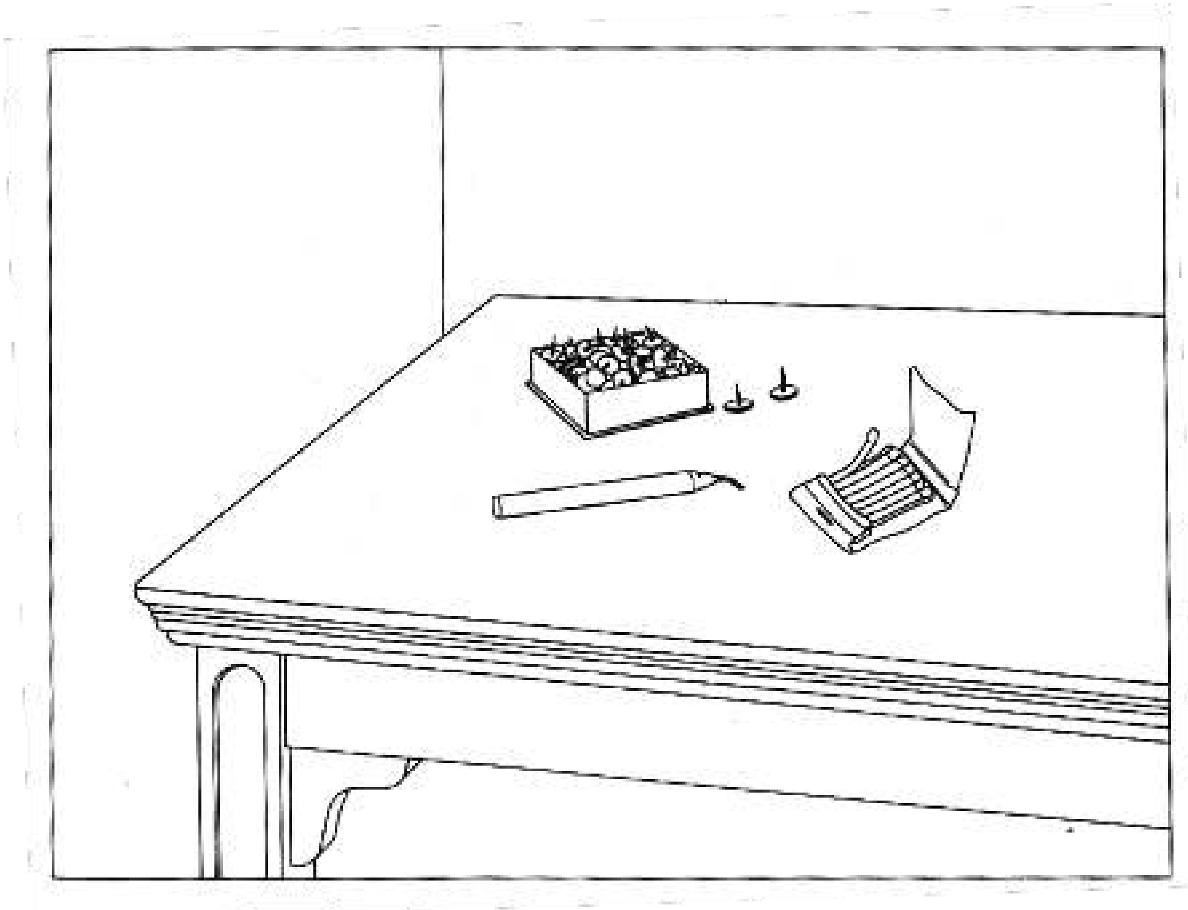


Functional Fixedness



Maier's (1931) two-string problem

Duncker's problem: support a candle on a door



A box of tacks, some matches, and a candle

People fixate on the box's function as a container

Why people get stuck solving problems

- **Functional Fixedness**
  - Subjects who utilize an object for a particular function will have more trouble in a problem-solving situation that requires a new and dissimilar function for the object
  - Young children suffer less from functional fixedness
    - ✓ Less experience might help...

Kohler (1945): monkey and banana problem

Kohler observed that chimpanzees appeared to have an insight into the problem before solving it

Insight

- Seemingly sudden understanding of a problem
- Often involves conceptualizing a problem in a totally different way (e.g. six stick problem, overcoming functional fixedness)
- How can we distinguish between problems requiring insight and problems requiring noninsightful problem solving?

Evidence for concept of insight:

Metcalfe and Weibe (1987) experiment

- Noninsight problem (algebra):
  - factor 16y^2 – 40yz + 25z^2

- Insight problem (nonroutine):
  - A prisoner was attempting escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How could he have done this?

Results (1)

- First result: subjects “feelings of knowing” (beforehand) only predicted eventual success of solving the problem for noninsight problems.

- At 15 seconds intervals, ss. rated how close they felt to solving the problem:
  - 1=cold (nowhere close to solution)
  - ….
  - 7=hot (problem is virtually solved)

Results (2)

Number of times a particular warmth rating was given

Expertise

Developing Expertise

- What are differences between novices and experts?
- How to become an expert?
- Examples of expertise:
  - Memory experts who can memorize long random strings of digits or letters
  - Mental calculation experts
  - Medical expertise
  - Chess experts

See anything unusual?

- Experts need only a few seconds to see what is wrong (or what isn't)
- Experts perceive large meaningful patterns in their domain

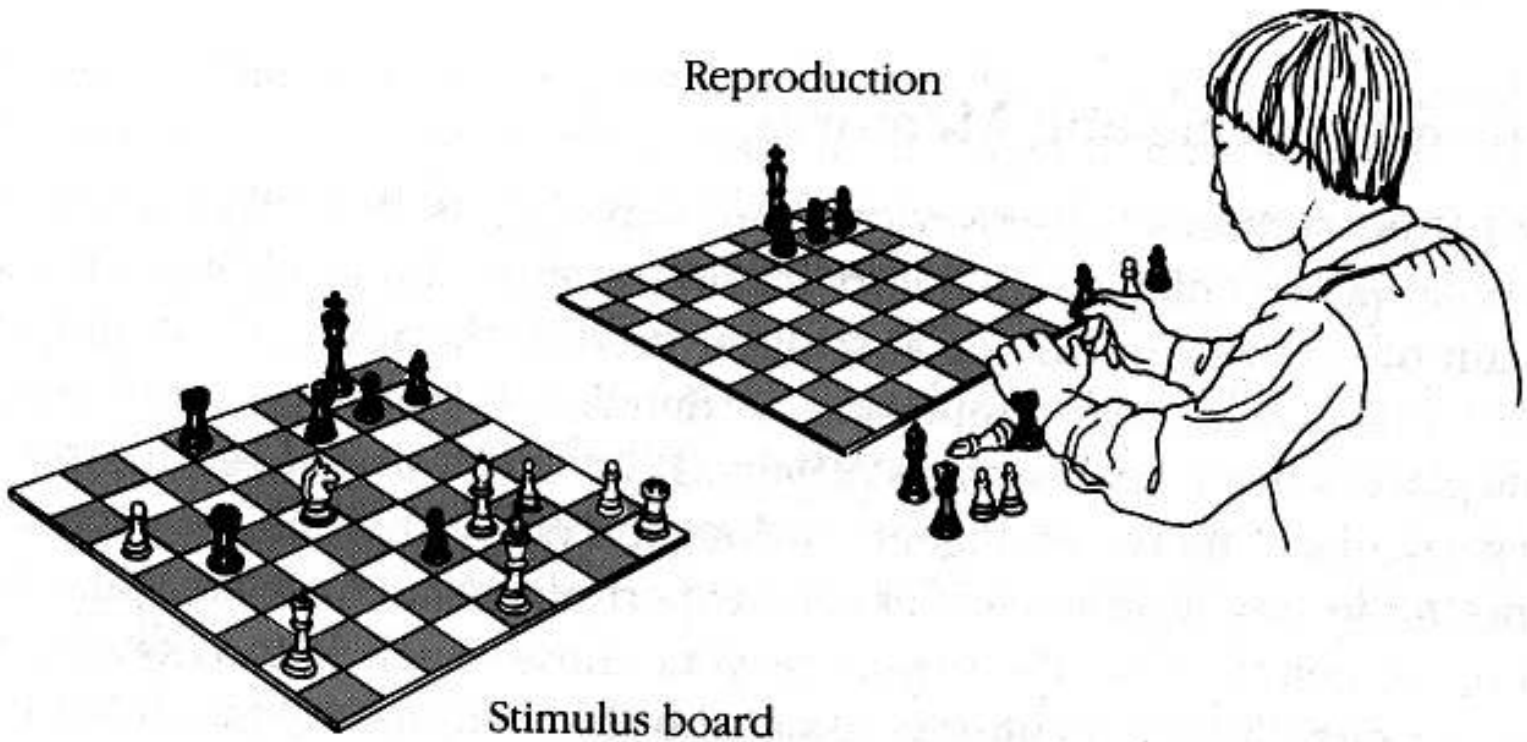
(normal)

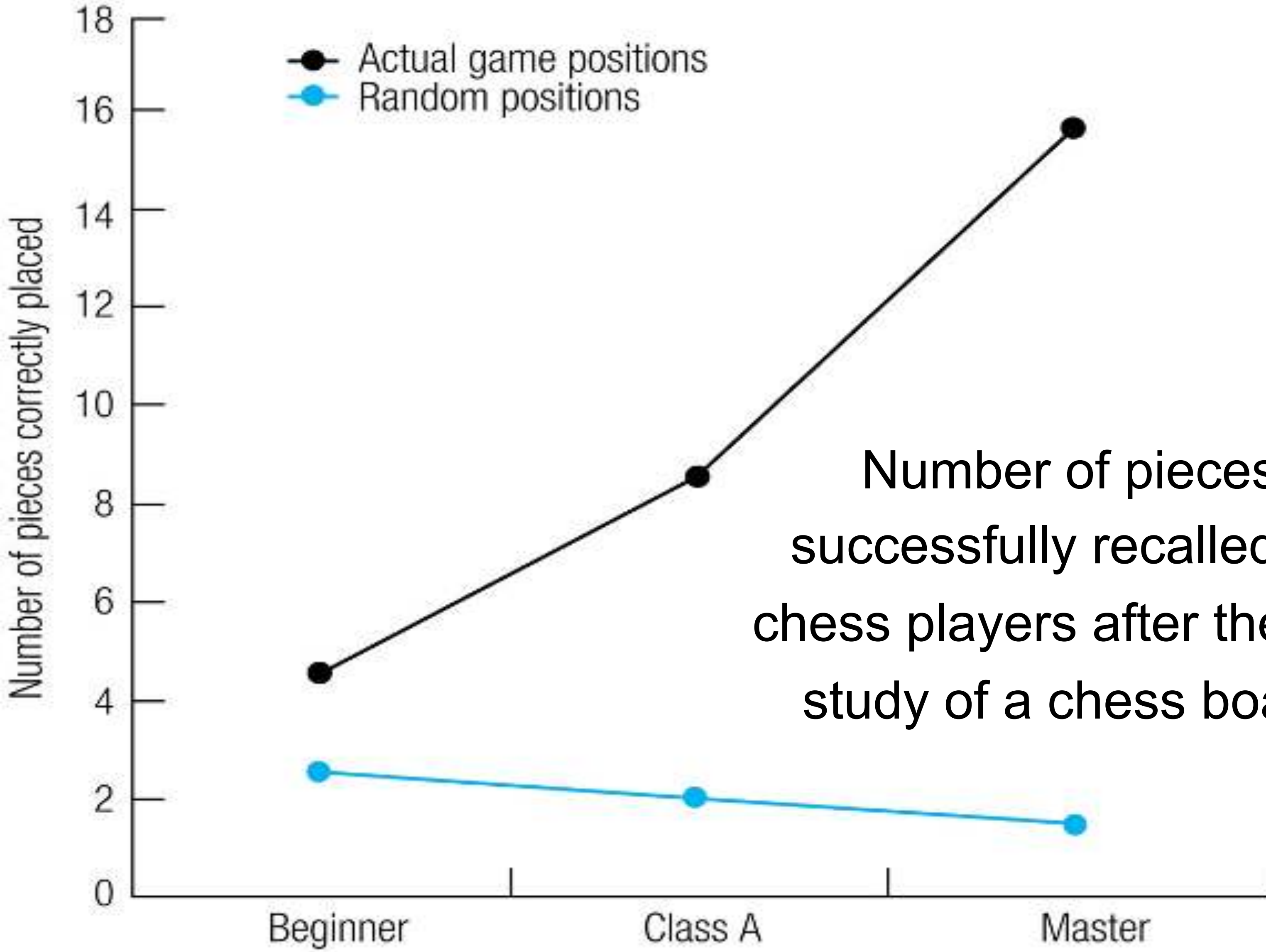
(collapse of the upper right lobe, upper left in picture)

Chess Studies

- De Groot (1965)
- Instructed 5 chess grandmasters to think out loud
- Grandmasters only considered about 30 moves and only thought 6 moves ahead
- Not that different from novices. However, The 30 moves considered by a grandmaster are really good moves
- Masters rely on extensive experience: 50,000 patterns

Chase & Simon (1973)





Number of pieces successfully recalled by chess players after the first study of a chess board

Chase & Simon, 1973

Conclusion from Chase & Simon (1973)

- Chess masters only expert with real chess positions. They do not have better memory in general

- Expertise allows **chunking** of salient information to promote memory of good moves

- Experts organize knowledge differently – reflects a deep understanding

What makes an expert an expert?

- Talent? IQ? Practice? Genetic factors?

- Experts are masters mostly in their own domain; the skill does not cross into different domains

- Study exceptional feats:
  - Memory experts
  - Chess experts
  - Musicians
  - Athletes

Voss et al., 1983

10 Year Rule

- 10 years of **deliberate practice** needed to attain an international level

- **Deliberate practice**: practice that is highly motivated and involves careful self-monitoring

- Master chess players spend 10,000 – 20,000 hours playing

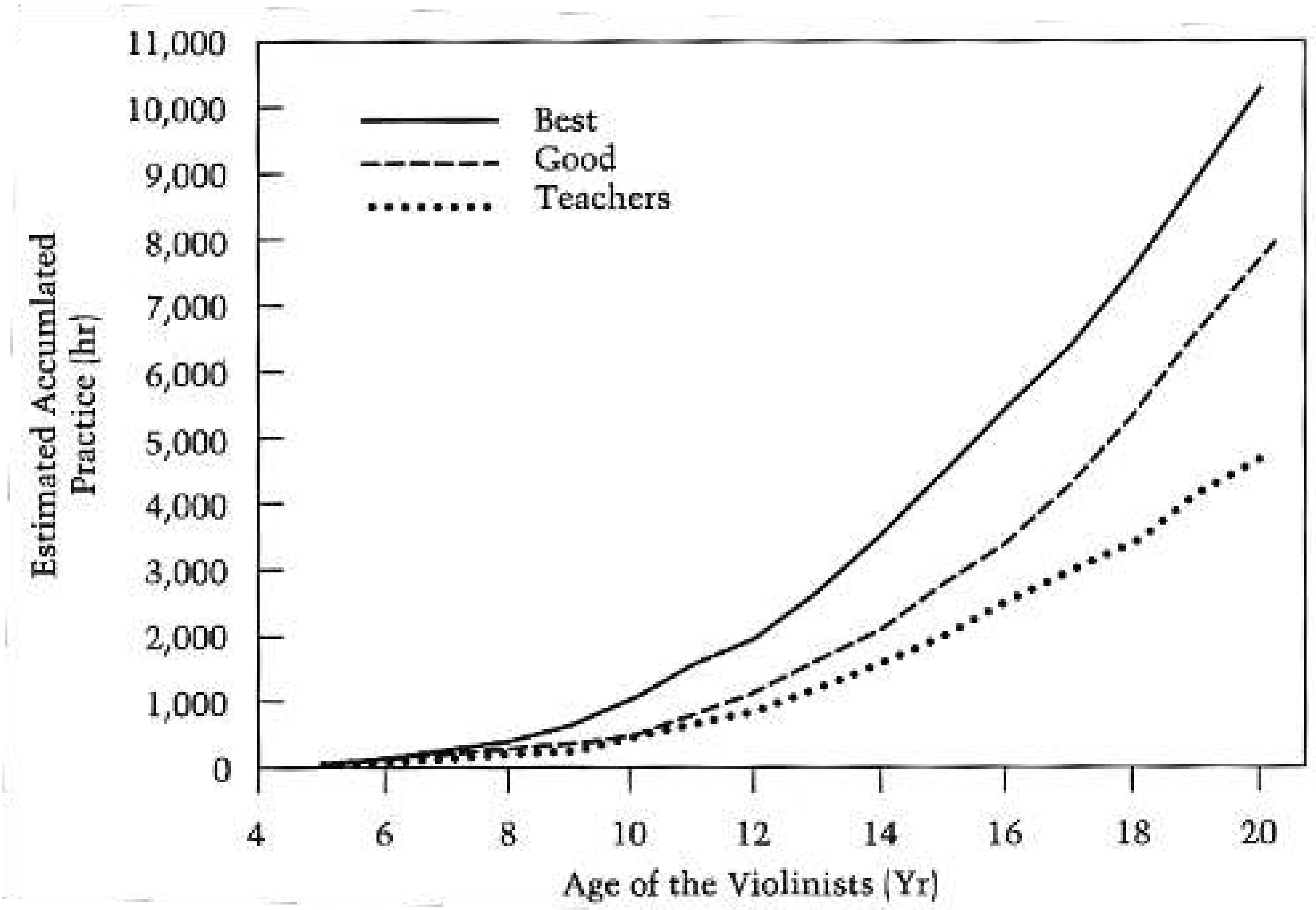
What about talent?

- Maybe exceptional performance in some area can be explained by talent – an innate predisposition that predetermines performance in a domain

- Anders Ericsson et al
  - disagree that concept of talent is useful or explains anything

    (genius is 90% perspiration and 10% inspiration)
  - this is controversial!

Difference between good and exceptional musicians is related to the amount of practice



Graph from Ericsson et al. (1996) showing the cumulative amount of practice by two groups of aspiring musical performers (experts and good violinists) and those who planned to teach music

Similarly, Anderson, Reder, and Simon (1998) note that one of the major reasons for the higher achievement in mathematics of students in Asian countries is that those students spend twice as much time practicing mathematics